# Microwaves

**Series 1**

**Problem 1**
A RADAR emits a 400kW signal at 8.5 GHz towards Ganymede, one of Jupiter's satellites. Its antenna has a gain of 72 dB. The reflected signal reaches the Earth 1 hour and 7 minutes after emission. Ganymede's diameter is of 2635 km and it reflects 12% of the power that a metallic sphere would reflect. What is the reflected power received by the RADAR ?

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The distance is here given by the traveling time, it is thus equal to:

$$\frac{c_o t_{ar}}{2} = 602.6 \ 10^9 \ \ m$$

The equivalent RADAR surface of a sphere of 2635 km in diameter reflecting 12% of what a conductive sphere would reflect is given by :

$$\sigma = 0.12 \pi a^2 = 654.4 \ \ 10^9 \ \ m^2$$

A gain G=72 dB corresponds to g=15.85 $10^6$, and the wavelength at 8.5 GHz is 0.03529 m. Introducing these values in the RADAR equation, we write :

$$P_r = \frac{400 \cdot 10^3 \left(15.85 \cdot 10^6 \cdot 0.03529\right)^2 652 \cdot 10^9}{(4\pi)^3 \left(603 \cdot 10^9\right)^4} = 3.1306 \cdot 10^{-22} \ \ W = -185 \ \ dBm$$

Which is very small indeed !!

**Problem 2**

A Doppler RADAR emits a signal at 35GHz. Which is the Doppler frequency produced by a car circulating with a speed of 140 km/h towards the RADAR?

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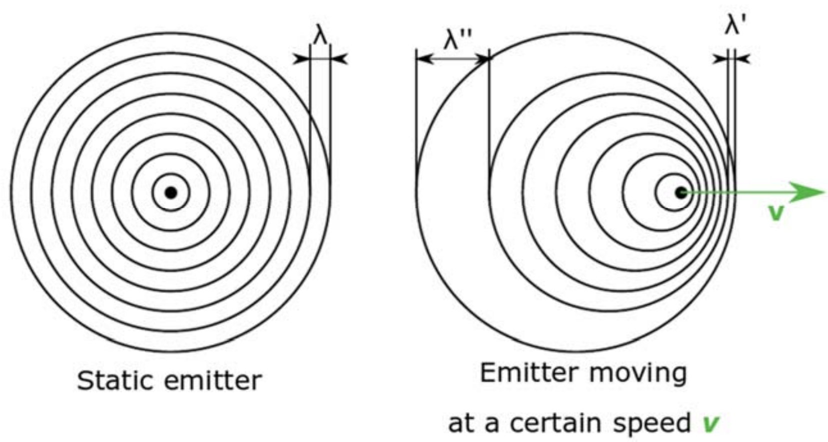
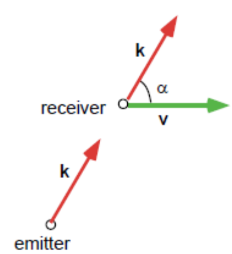
One should keep in mind that for each of these scenarios the frequency varies depending if the emitter is moving towards the receiver or away from it. This phenomenon is illustrated in a simplified way for an acoustic wave on the following picture.




To understand the problem one can split it in two scenarios.

- When the RADAR emits the signal towards the car.

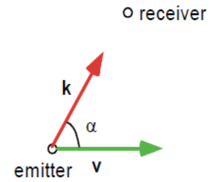
$$f_r = f_o\left(1 + \frac{v}{c}\cos\alpha\right)$$





- When the signal is reflected from the car.

$$f_r = \frac{f_o}{\left(1 + \frac{v}{c}\cos\alpha\right)}$$





From the previous formulas the Doppler equation is obtained, which directly gives the Doppler frequency:

$$f_D = \frac{2 f v_r}{c_0} = 9.074 \quad kHz$$

**Problem 3**

At which maximum latitude can we receive the signal of a geostationary satellite having a

a) grazing incidence (signals comes from the horizon and is parallel to the ground)
b) an elevation angle of 10° (signals comes from an elevation of 10° over the horizon)

The curvature effect of the atmosphere is neglected

Hint: make a drawing of the geometry of the problem

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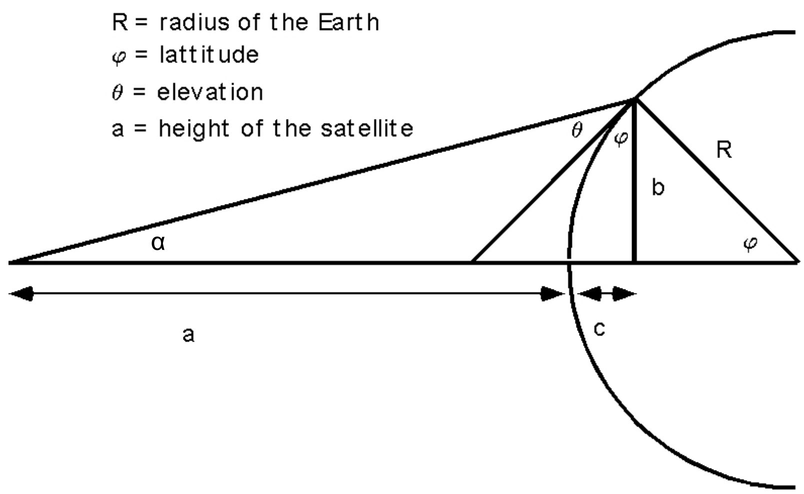
This is in fact a trigonometry problem, as shown in the following figure:




The theorem of the sinus yields:

$$\frac{\sin\alpha}{R} = \frac{\sin(90° + \theta)}{a + R}$$

From which:

For an incidence of θ=0°, the latitude is φ=81.3°
For an incidence of θ=10°, the latitude is φ=71.4°

**Problem 4**
Can this expression represent an induction field? Why?

$$\mathbf{B} = \frac{B_o}{a^2}\left(\mathbf{e_x}xz + \mathbf{e_y}yz + \mathbf{e_z}xy\right)$$

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We take the divergence of **B**:

$$\nabla \cdot \mathbf{B} = \frac{B_o}{a^2}\nabla.\left(\mathbf{e_x}xz + \mathbf{e_y}yz + \mathbf{e_z}xy\right) = \frac{B_o}{a^2}\left(\frac{\partial xz}{\partial x} + \frac{\partial yz}{\partial y} + \frac{\partial xy}{\partial z}\right) = \frac{B_o}{a^2}2z \neq 0$$

This expression being not equal to zero, B cannot be an induction field.